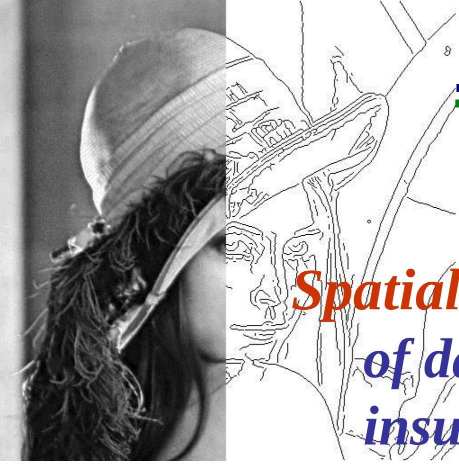


Sampling and Spatial Resolution

Spatial Aliasing Problem: ***Spatial aliasing*** ***is insufficient sampling of data along the space axis, which occurs because of the insufficient spatial resolution of the acquired image.***

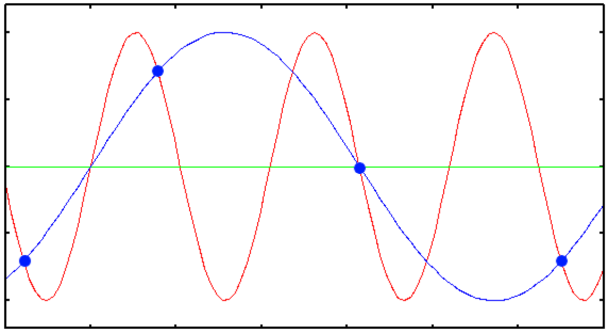
- It is recommended to sample the image at a rate close to the ideal sampling rate (Nyquist Rate). Images obtained with sampling distances larger than those established by this rate suffer from undersampling.

- The Nyquist Rate is defined as twice the bandwidth of the continuous-time signal. It should be noted that the sampling frequency must be strictly greater than the Nyquist Rate of the signal to achieve unambiguous representation of the signal.

- The Critical Sampling Distance is the Sampling Distance corresponding to the Nyquist Rate.

Sampling and Spatial Resolution

***Spatial Aliasing Problem**: In statistics, signal processing, and related disciplines, aliasing is an effect that causes different continuous signals to become indistinguishable (or aliases of one another) when sampled. When this happens, the original signal cannot be uniquely reconstructed from the sampled signal.*

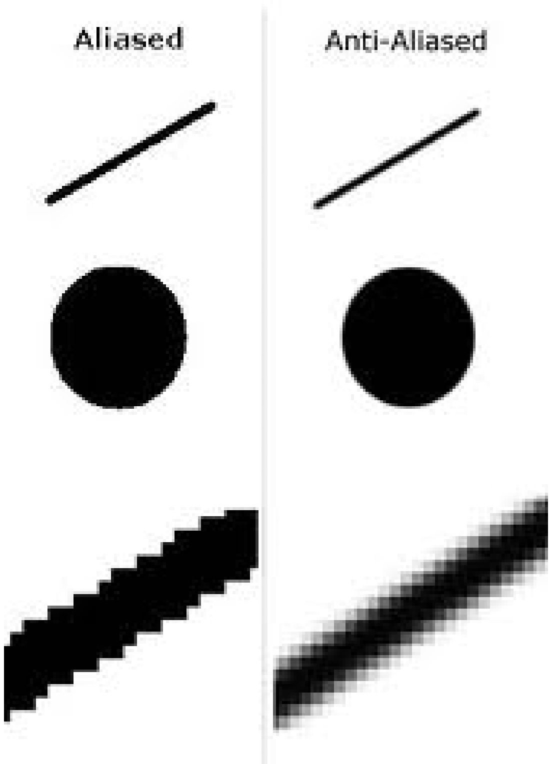


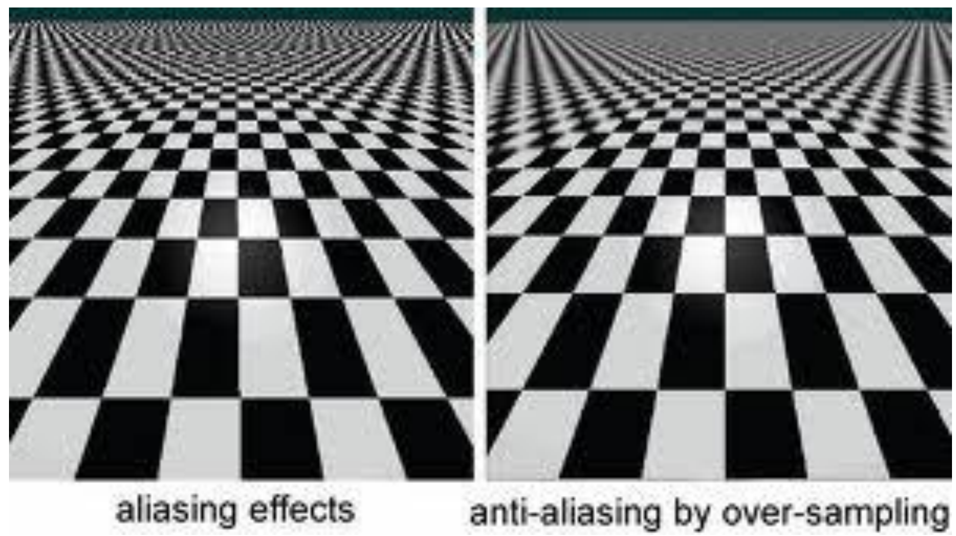
- **aliasing of a *undersampled* 1D sinusoidal signal**

- **aliasing of a *undersampled* 2D image**

Sampling and Spatial Resolution

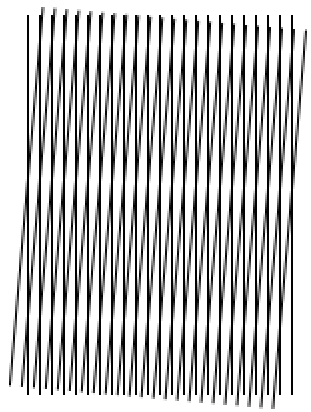
Spatial Aliasing Problem**: *In statistics, signal processing, and related disciplines, aliasing is an effect that causes different continuous signals to become indistinguishable (or aliases of one another) when sampled. When this happens, the original signal cannot be uniquely reconstructed from the sampled signal.***



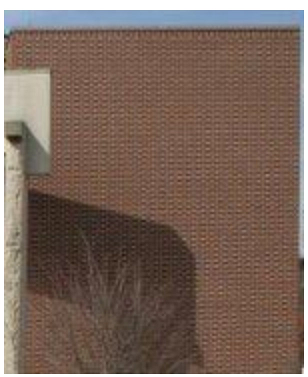


Sampling and Spatial Resolution

***Spatial Aliasing Problem**: Aliasing may give rise to moiré patterns (when the original image is finely textured) or jagged outlines (when the original has sharp contrasting edges, e.g. screen fonts).*

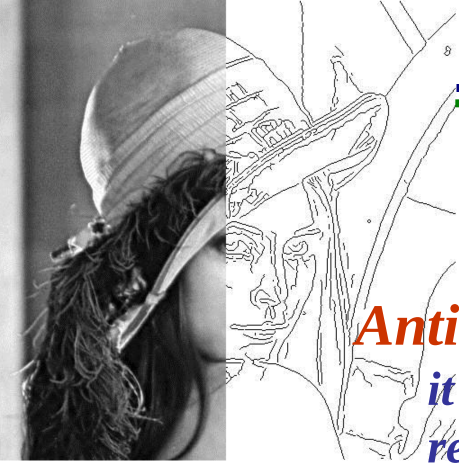


i.e. a *moiré pattern*, formed by two sets of parallel lines, one set inclined at an angle of 5° to the other



Original Image

A *moiré pattern* formed by incorrectly *downsampling* the image



Sampling and Spatial Resolution

***Anti-aliasing** refers to the low pass filters (LPF) applied to an image before it is downsampled (downscaled) to avoid → Aliasing Artifacts. The filters remove high-frequency content from the original image reducing its Band Width, and therefore the lower Sampling Density in the downscaled image is still above the Critical Sampling Distance.*

Aliasing problem of a downsampled image

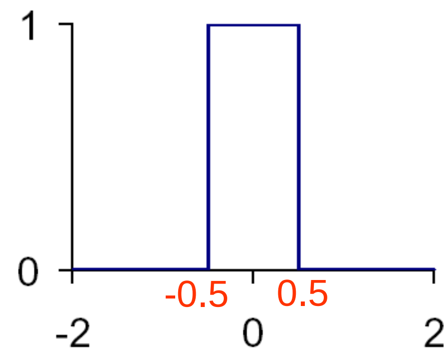
Aliasing problem avoided by using band limiting Low Pass Filter.

Up-Sampling - Image Zooming

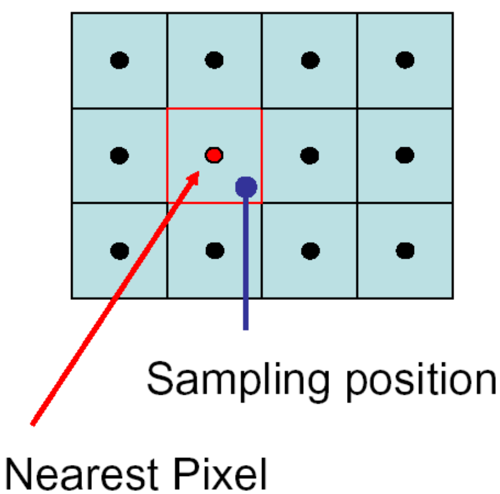
Nearest Neighbor Interpolation:

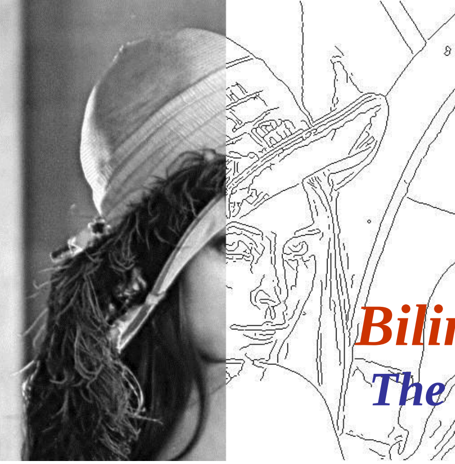
The Interpolation kernel for the nearest neighbor interpolation is defined by:

$$h(x) = \begin{cases} 1 & 0 \le |x| < 0.5 \\ 0 & 0.5 \le |x| \end{cases}$$



Interpolation Kernel



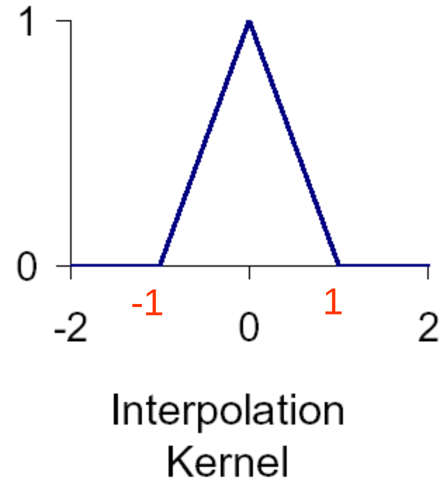


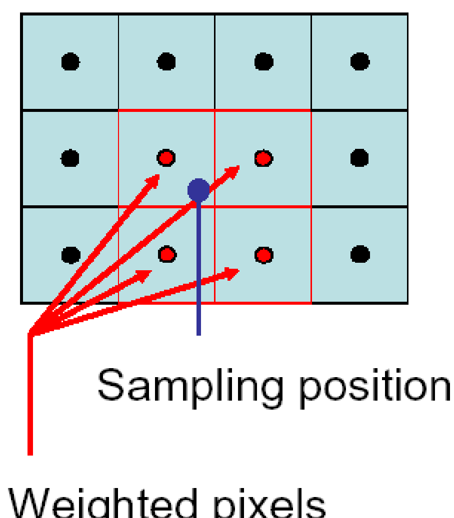
Up-Sampling - Image Zooming

Bilinear Interpolation:

The Interpolation kernel for the bilinear interpolation is defined by:

$$h(x) = \begin{cases} 1 - |x| & 0 \le |x| < 1 \\ 0 & 1 \le |x| \end{cases}$$



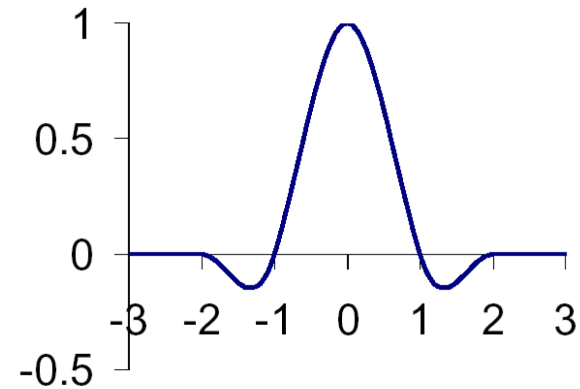


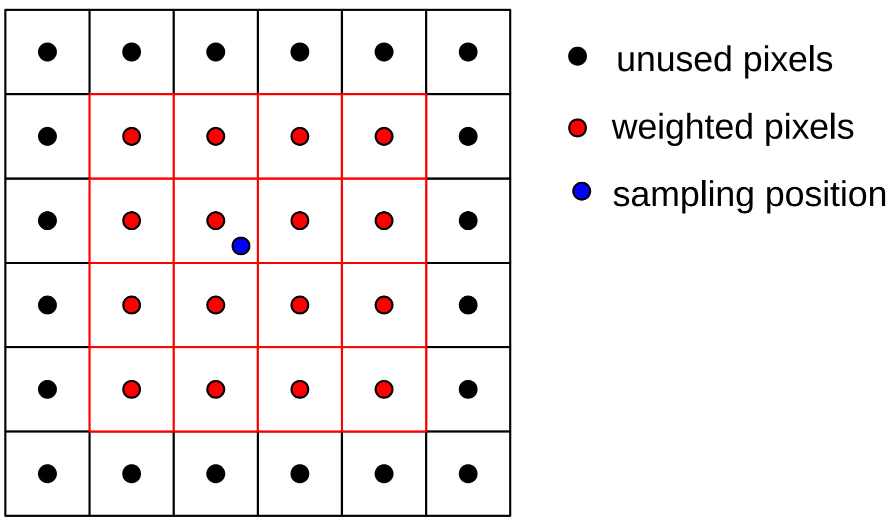
Up-Sampling - Image Zooming

Bicubic Interpolation:

The Interpolation kernel for the bicubic interpolation is defined by:

$$h(x) = \begin{cases} (a+2)|x|^3 - (a+3)|x|^2 + 1 & 0 \le |x| < 1 \\ a|x|^3 - 5a|x|^2 + 8a|x| - 4a & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases}$$



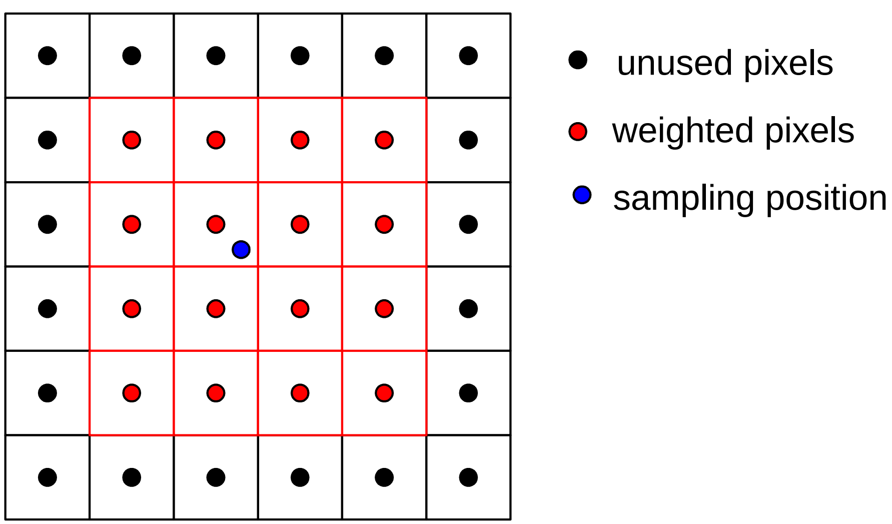


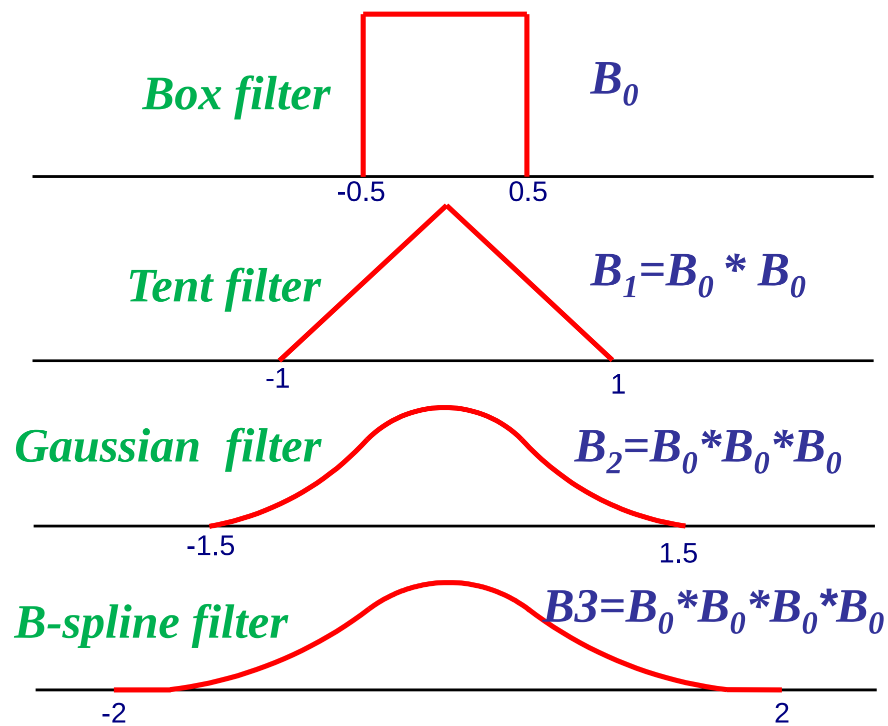
Up-Sampling - Image Zooming

Spline Interpolation:

There are many spline interpolation kernels. Among them the Interpolation kernel for the B-spline interpolation is defined by and B-spline of degree n is derived through n convolutions of the box filter, B_0.

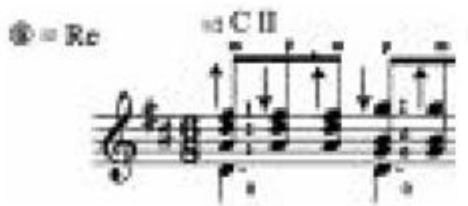
$$h(x) = \frac{1}{6}\begin{cases} 3|x|^3 - 6|x|^2 + 4 & 0 \le |x| < 1 \\ -|x|^3 + 6|x|^2 - 12|x| + 8 & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases}$$





Up-Sampling - Image Zooming

Comparing Interpolation Techniques:



Nearest Neighbor

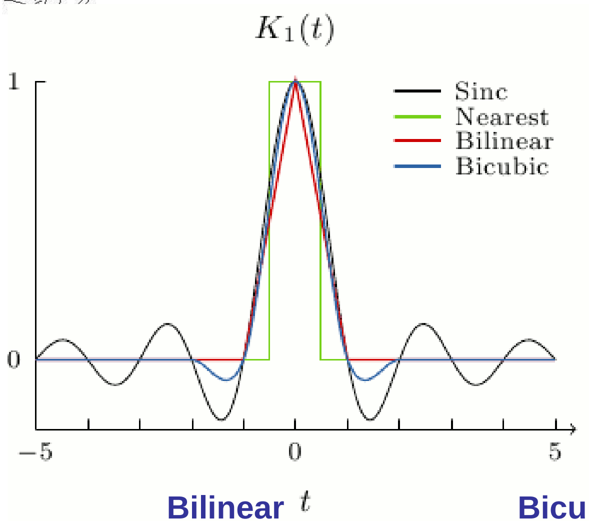
Bilinear

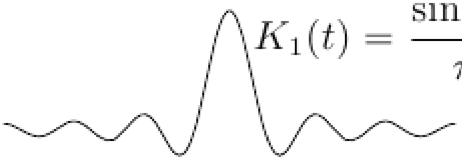
Bicubic

B-spline

Up-Sampling - Image Zooming

Comparing Interpolation Techniques: *The Sinc function can also be used as the interpolation kernel.*

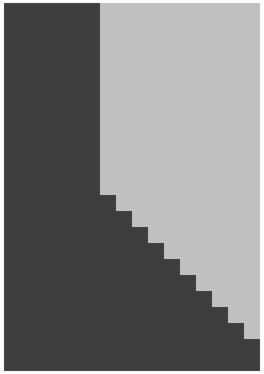




$$K_1(t) = \frac{\sin(\pi t)}{\pi t}$$

The *sinc* function

Nearest



Bilinear

Bicubic

B-spline

Sinc

Image Enhancement

Definition: Image enhancement is the process that improves the quality of the image for a specific application.

Image Enhancement Methods

Spatial Domain methods: The term spatial domain refers to the 2-D image plane and the approaches manipulates the pixel of a given image for enhancement.

Frequency Domain methods: The methods in frequency domain manipulate the Fourier transform of a given image for enhancement.

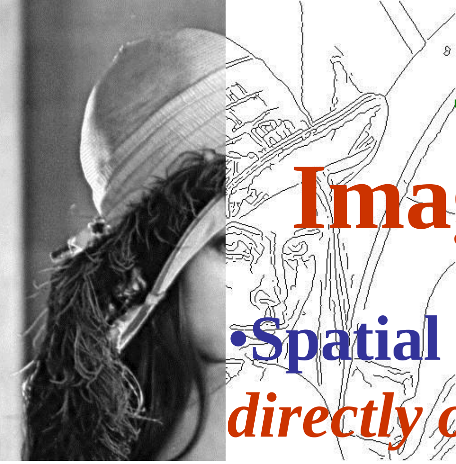


Image Enhancement in *Spatial Domain*

- **Spatial Domain methods are procedures that *operate directly on pixels*.**

- **A process in Spatial Domain can be denoted by:**

$$\boxed{g(x,y) = T[f(x,y)]}$$

- ***f(x,y)* is the input and *g(x,y)* is the output image.**

- ***T* is the *operator (Transformation function)* on *f*, defined over a *neighborhood* of *(x,y)*.**

A *neighborhood* over/about a pixel *(x,y)* uses a *square/rectangular subimage* centered at *(x,y)*.

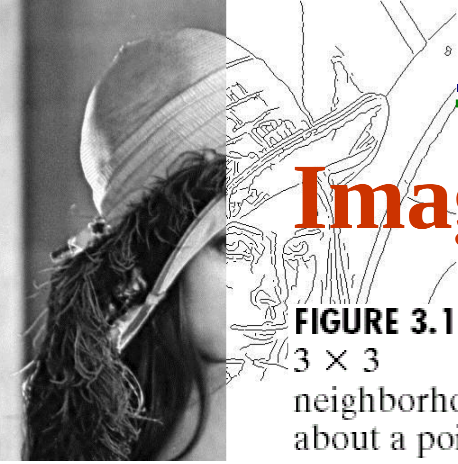
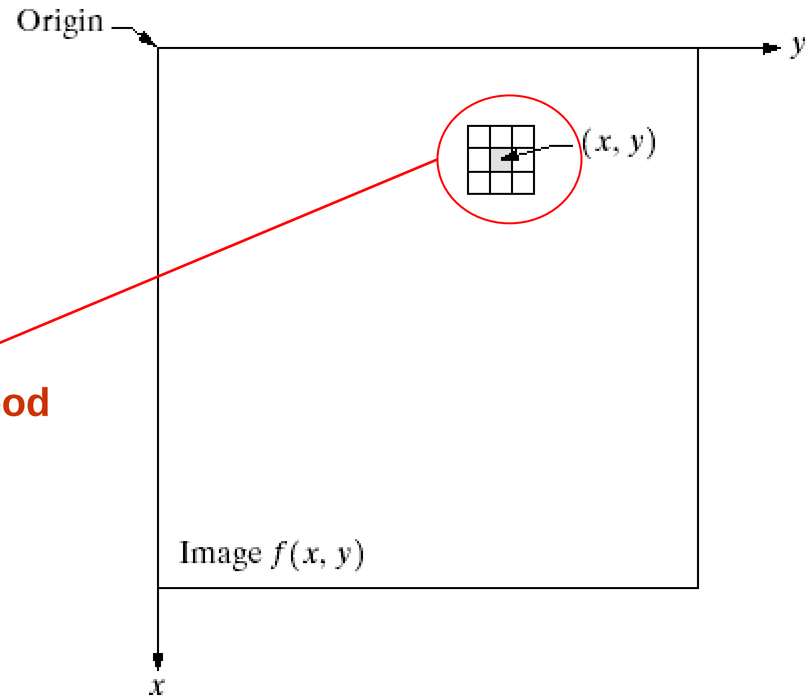


Image Enhancement in *Spatial Domain*

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



3x3 neighborhood

(mask)

- ***The center of the rectangular subimage is moved from the left top to the right bottom visiting all the pixels.***
- ***g(x,y) is calculated and output image is formed as each pixel is processed.***

Image Enhancement in *Spatial Domain*

*•If a 1x1 neighborhood is used for the transformation function, then the function is called the **gray-level transformation** function.*

$$s = T(r), \quad where, \quad s = g(x,y) \quad and \quad r = f(x,y)$$

Point Processing

*•Refers to the processing techniques where the enhancement **at any point** in an image depends **only** the **gray-level** at that point.*

Mask Processing / Filtering

*•**Mask** (also referred to as **filter/kernel/template/window**) is a 2-D array (i.e. 3x3) defined around a pixel, where the values of the **mask coefficients** determine the nature of the process.*

*•Image enhancement methods using mask approach is called **Mask Processing / Filtering**.*

Image Enhancement in *Spatial Domain*

Contrast Stretching

- ***Contrast stretching** (normalization) is a simple image enhancement technique that improves the contrast in an image by ' stretching' the range of intensity values it contains to span a desired range of values. Typically, it uses a **Linear Scaling Function**.*

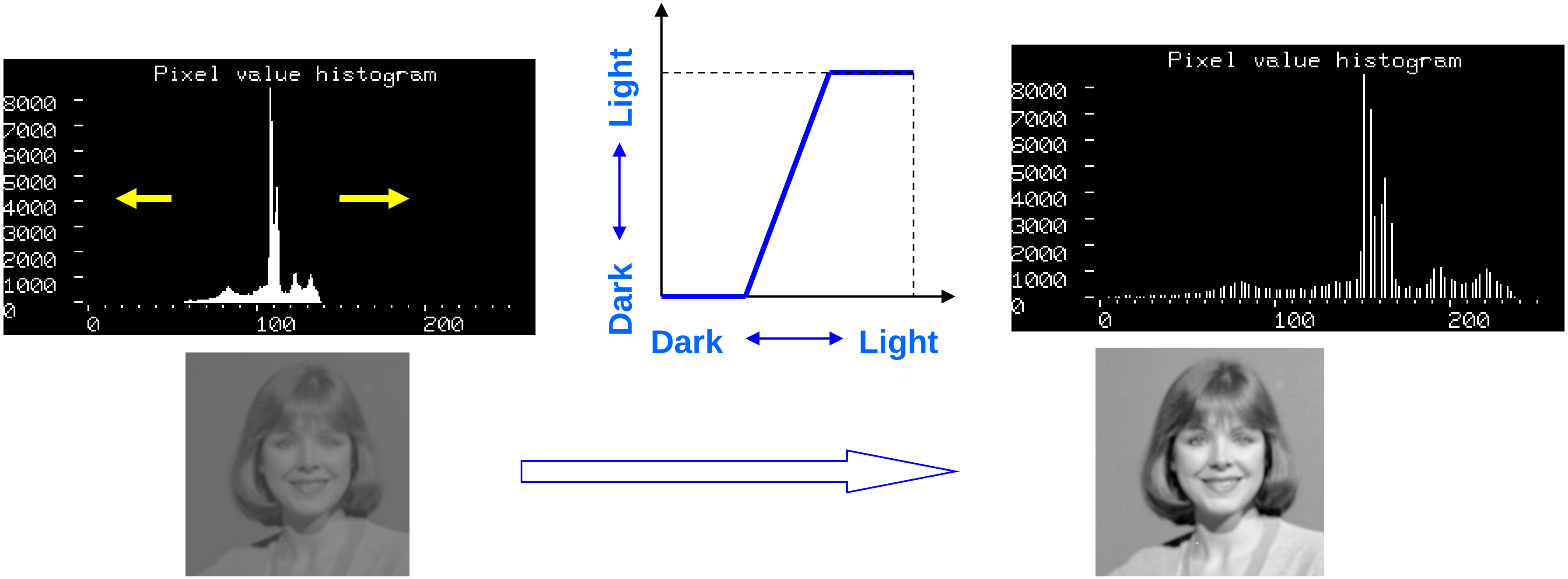


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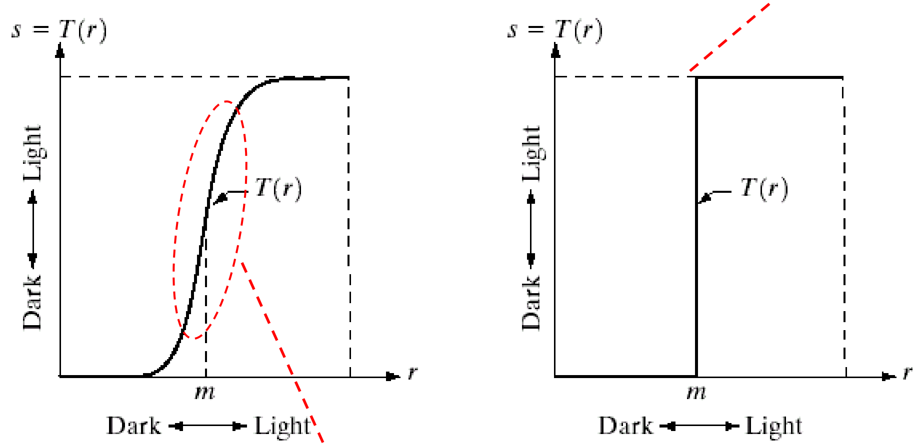
Image Enhancement in *Spatial Domain*

Contrast Stretching

Converts to black & white



a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Linear Part contributes to the contrast stretching

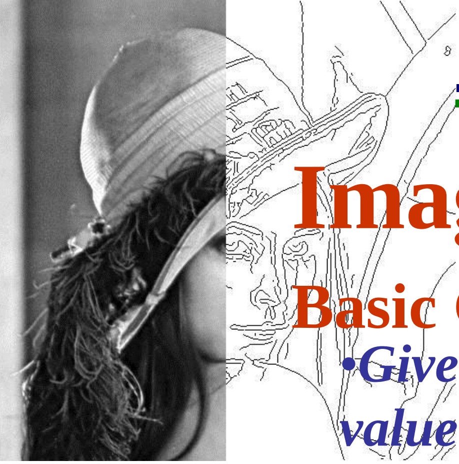
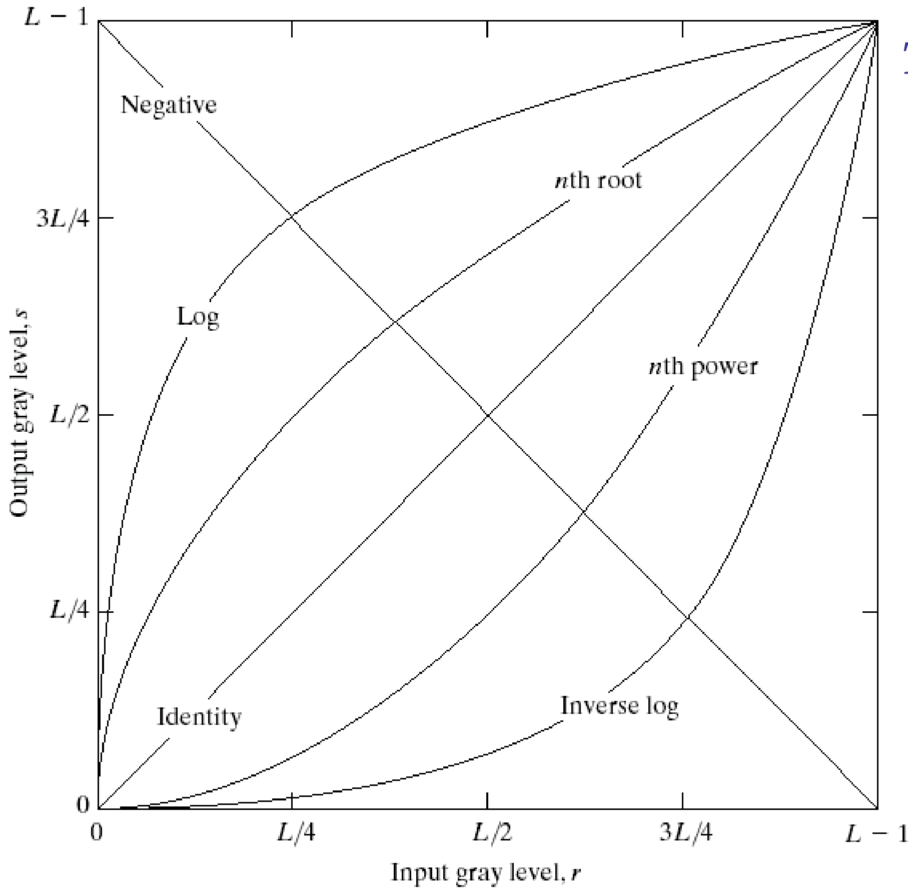


Image Enhancement in *Spatial Domain*

Basic Gray Level Transformations

- *Given a gray-level transformation function T(r) where each pixel value r is mapped to pixel value s.*

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



The transformation functions:
- *linear*
- *logarithmic (log/inverse-log)*
- *power-law (n^{th} power/n^{th} root)*

[0, L-1] is the ***gray-level range***

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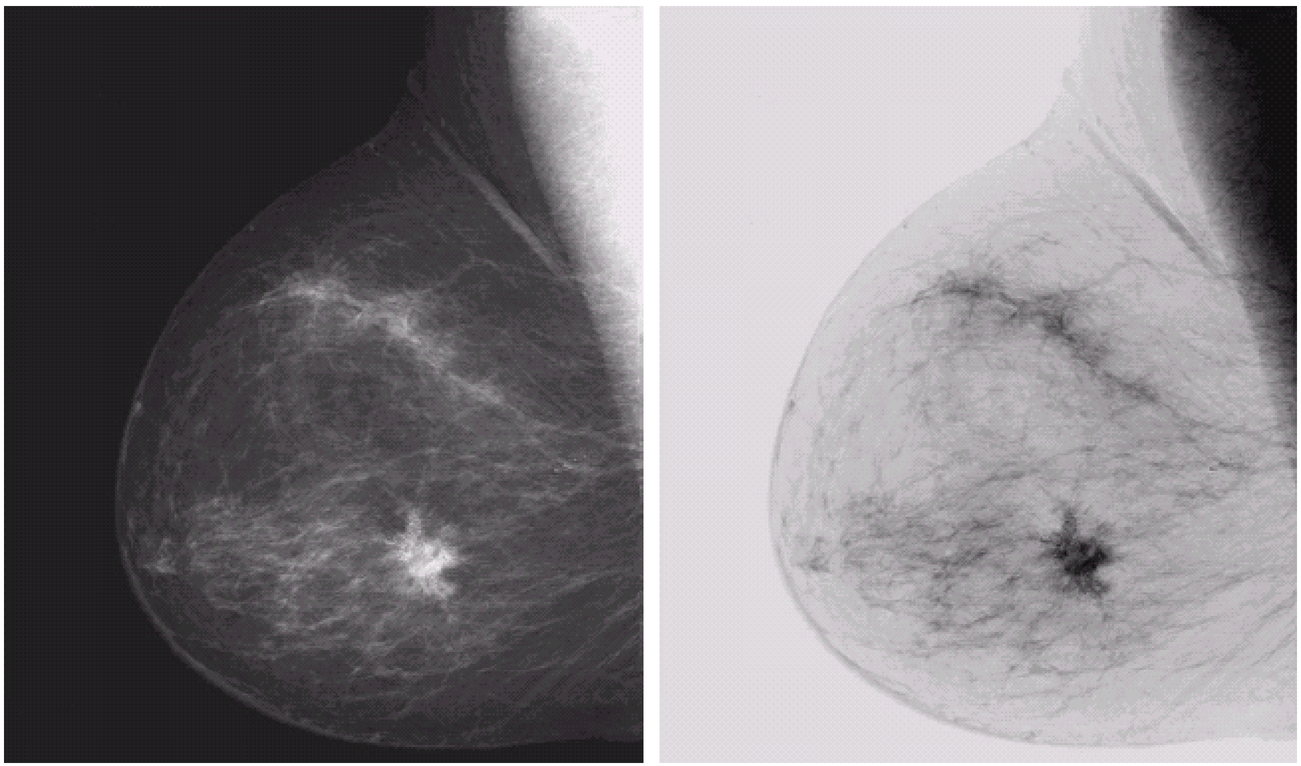
Image Enhancement in *Spatial Domain*

Image Negatives

- *The negative of an image with gray levels in the range [0,L-1] can be obtained by*

$$s = L - 1 - r$$



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

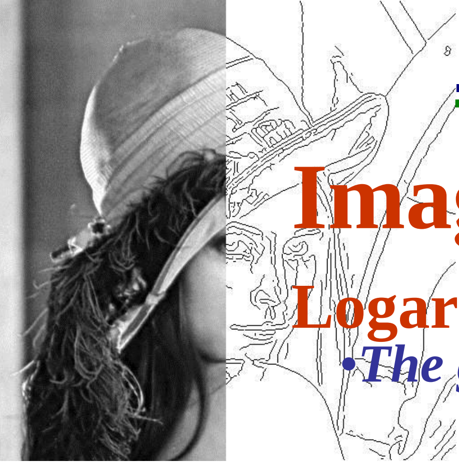


Image Enhancement in *Spatial Domain*

Logarithmic Transformations

• *The general form of the log transformations:*

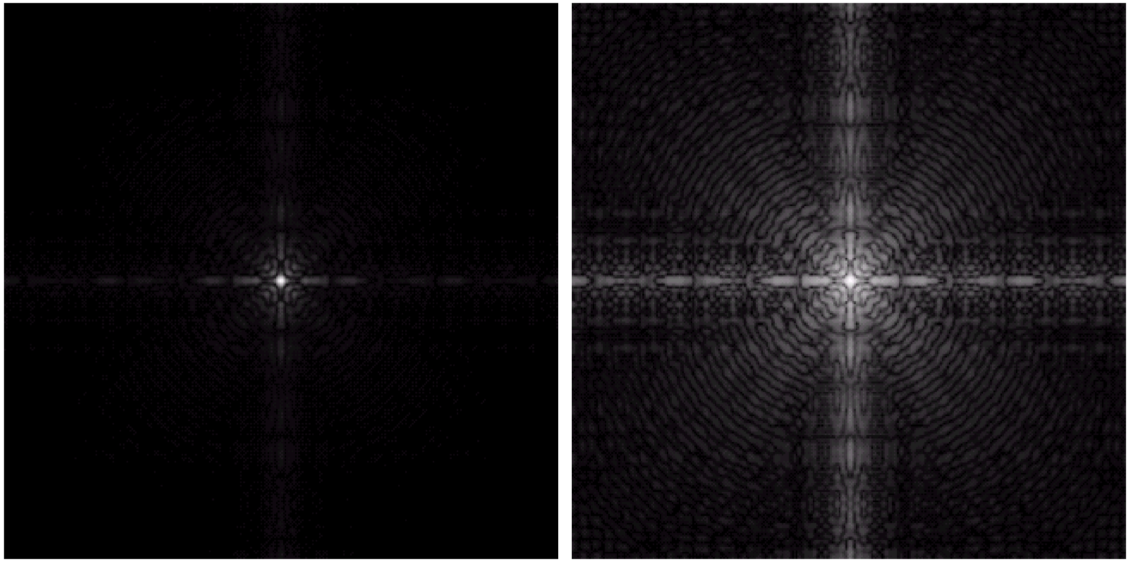
$$s = c\log(1+r), \quad c \text{ is a const. and } r \geq 0$$

• ***The Log Transforms compresses the dynamic range of images with large variations in pixel values.***

• ***Typical application is to change the large dynamic range of a Fourier Spectrum to a smaller range for a clearer inspection.***

a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



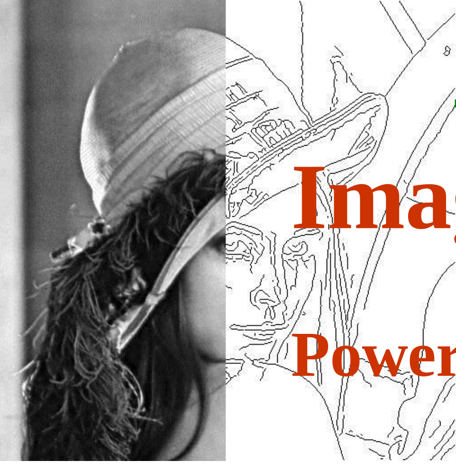


Image Enhancement in *Spatial Domain*

Power-Law Transformations

- *The general form of the Power-law transformation:*

$$s = cr^{\gamma}, \quad where \;\; c \, and \, \gamma \geq 0$$

- ***Different transformation curves are obtained by varying γ (gamma).***
- ***Many image capturing, printing and display devices use gamma correction which enhances the given images by power-law response phenomena.***

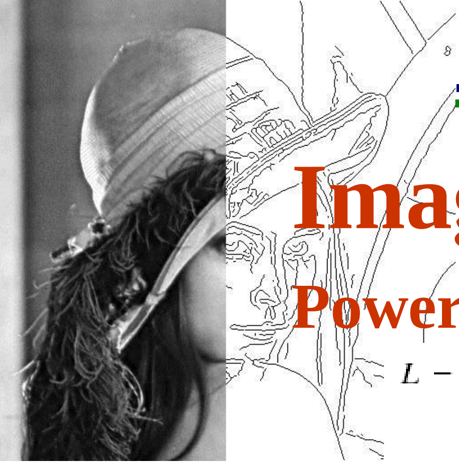


Image Enhancement in *Spatial Domain*

Power-Law Transformations – Gamma Correction

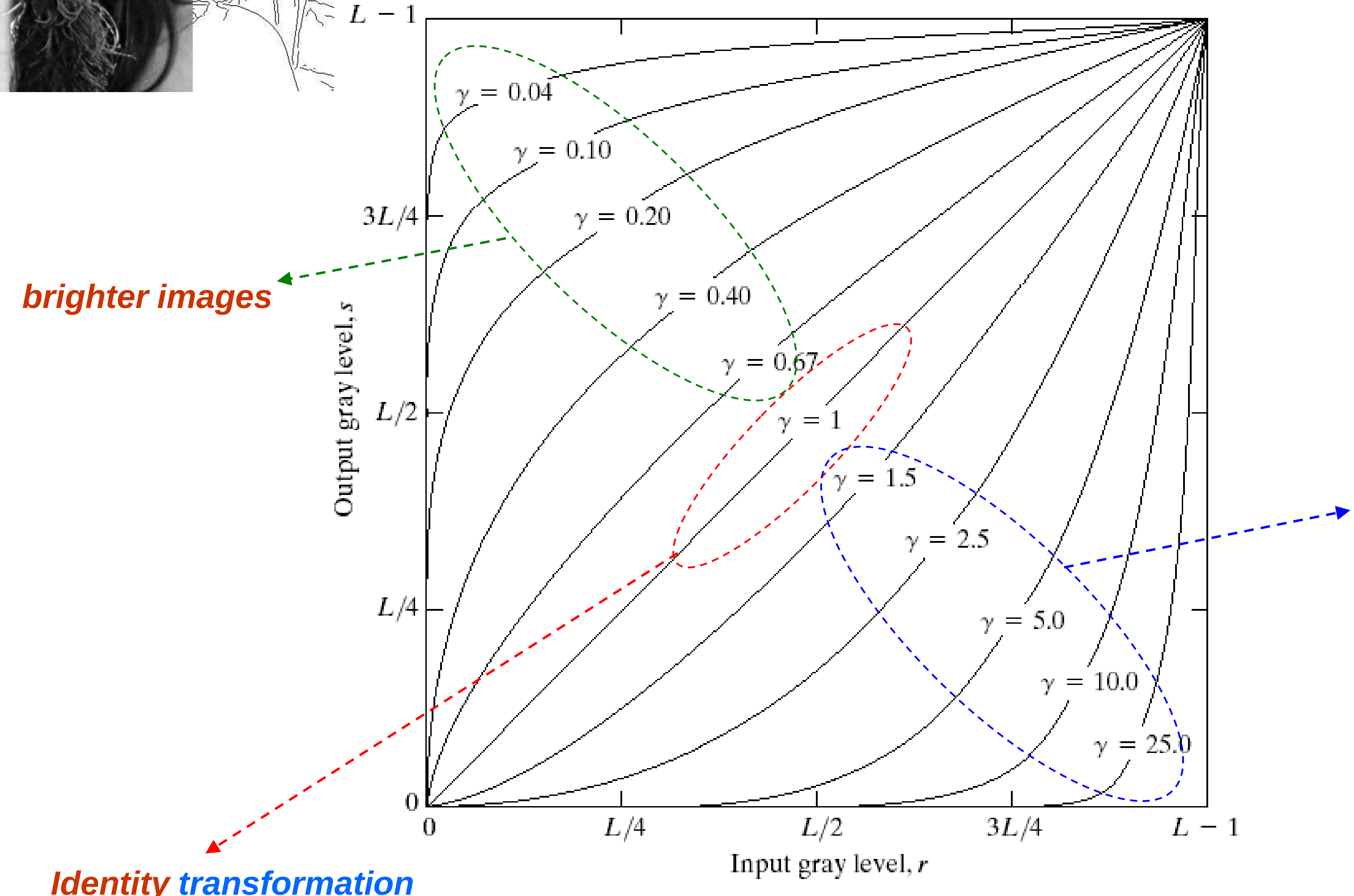


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

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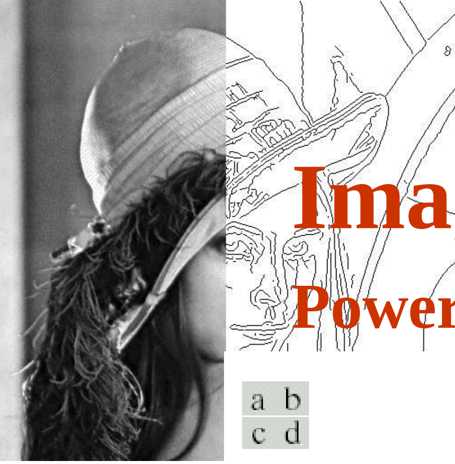
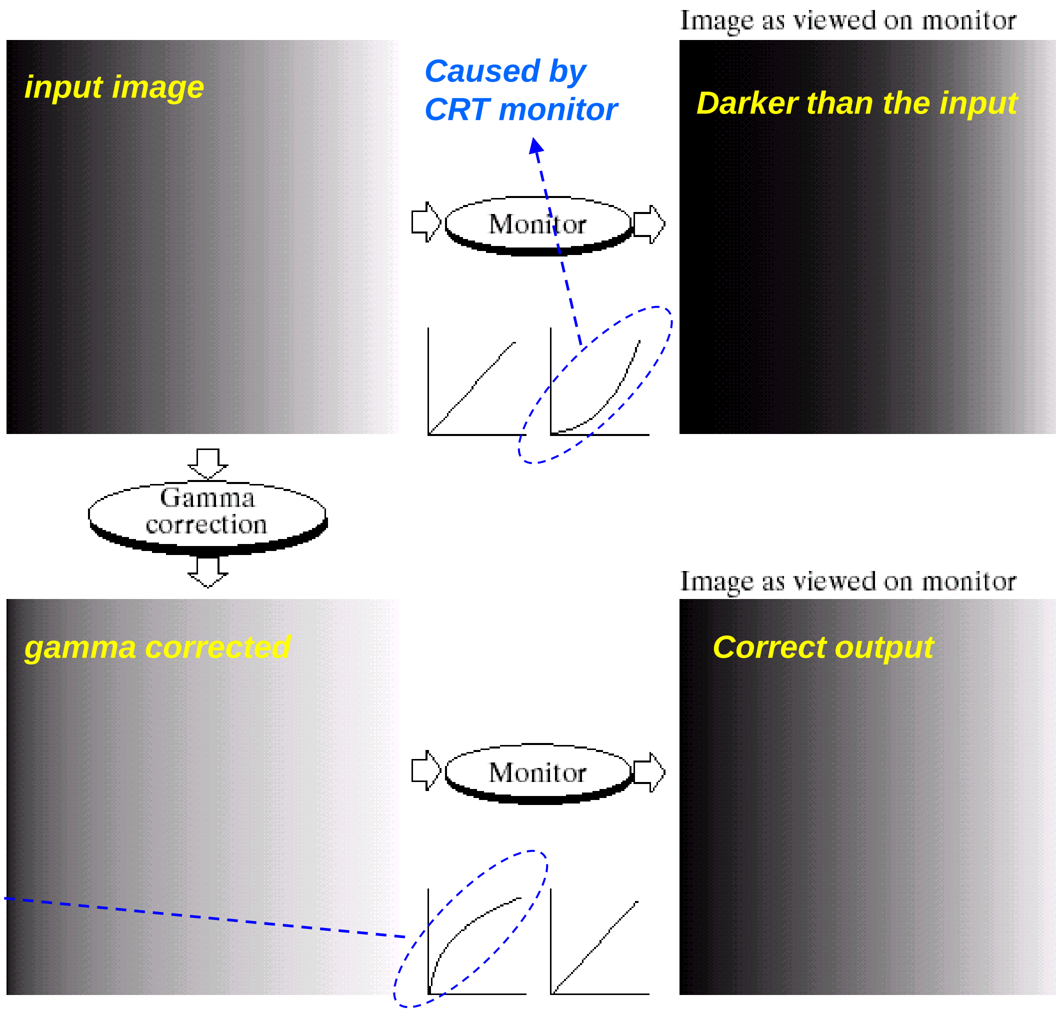


Image Enhancement in *Spatial Domain*

Power-Law Transformation – Gamma Correction



a b
c d

FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

gamma correction is used as pre-processing

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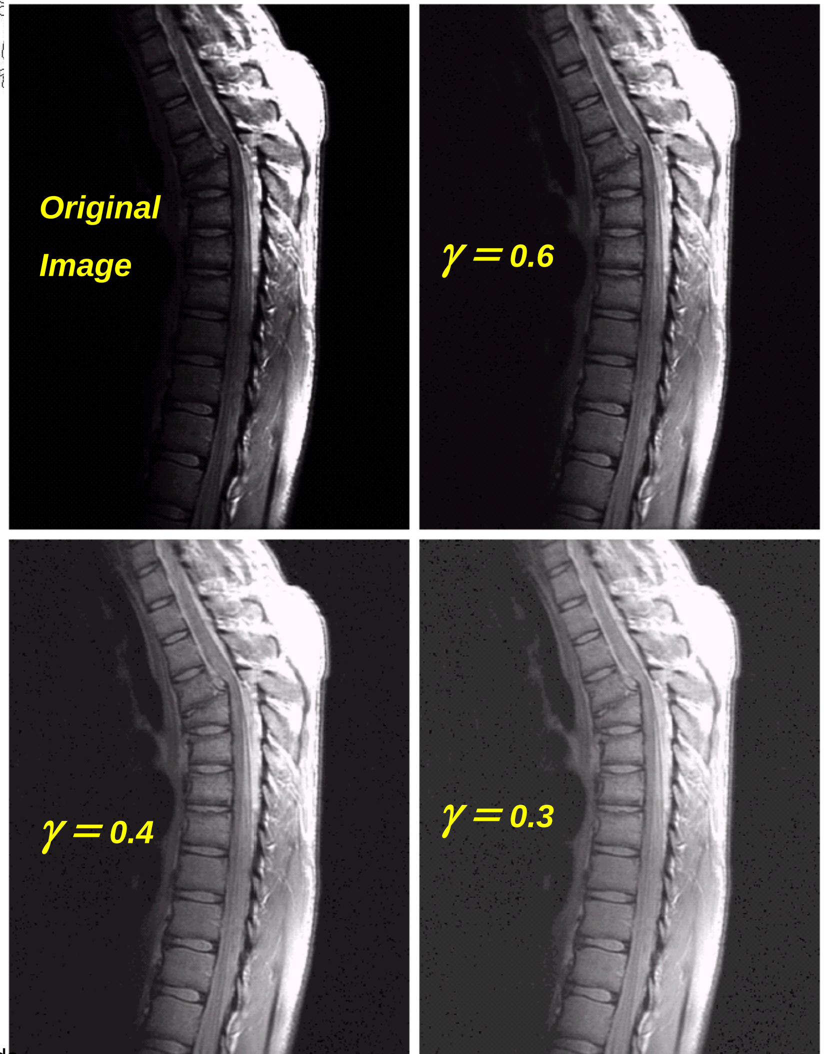
Image Enhancement in *Spatial Domain*

Power-Law Transformation – Gamma Correction

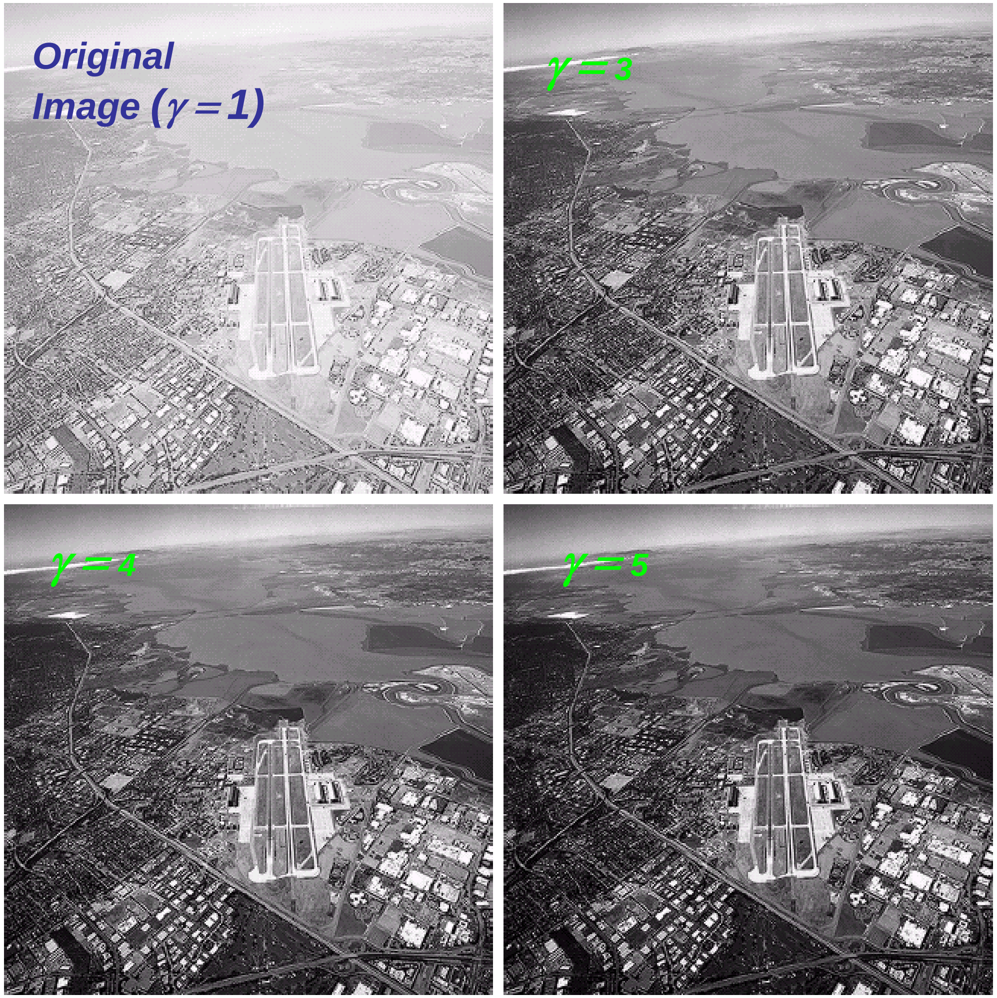


a	b
c	d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Image Enhancement in Spatial Domain

Power-Law Transformation – Gamma Correction



a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

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Image Enhancement in *Spatial Domain*

Piecewise-Linear Transformations – Contrast Stretching

•One of the simplest piecewise functions is the contract stretching. which is used to enhance the low contrast images.

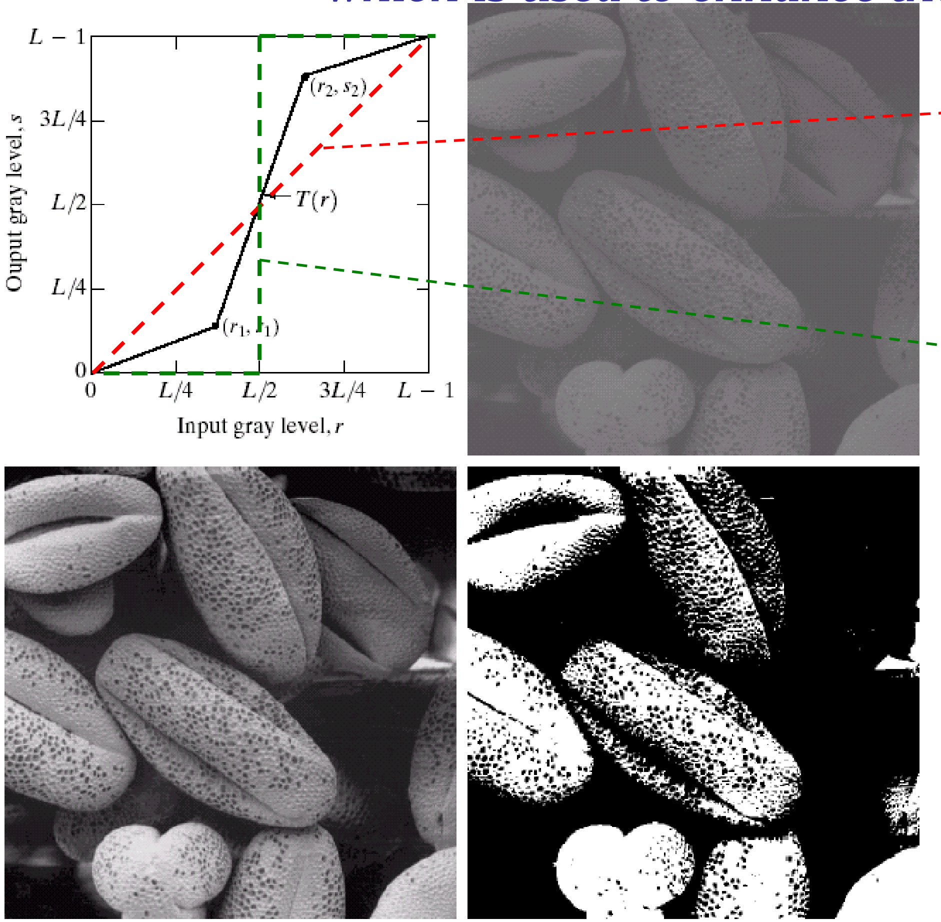

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

$s_1 = r_1$ and $s_2 = r_2$ ***(identity transformation)***

No Change in the image

$r_1 = r_2$ and $s_1 = 0$, $s_2 = L\text{-}1$ ***(thresholding function)***

Image converted to black & white

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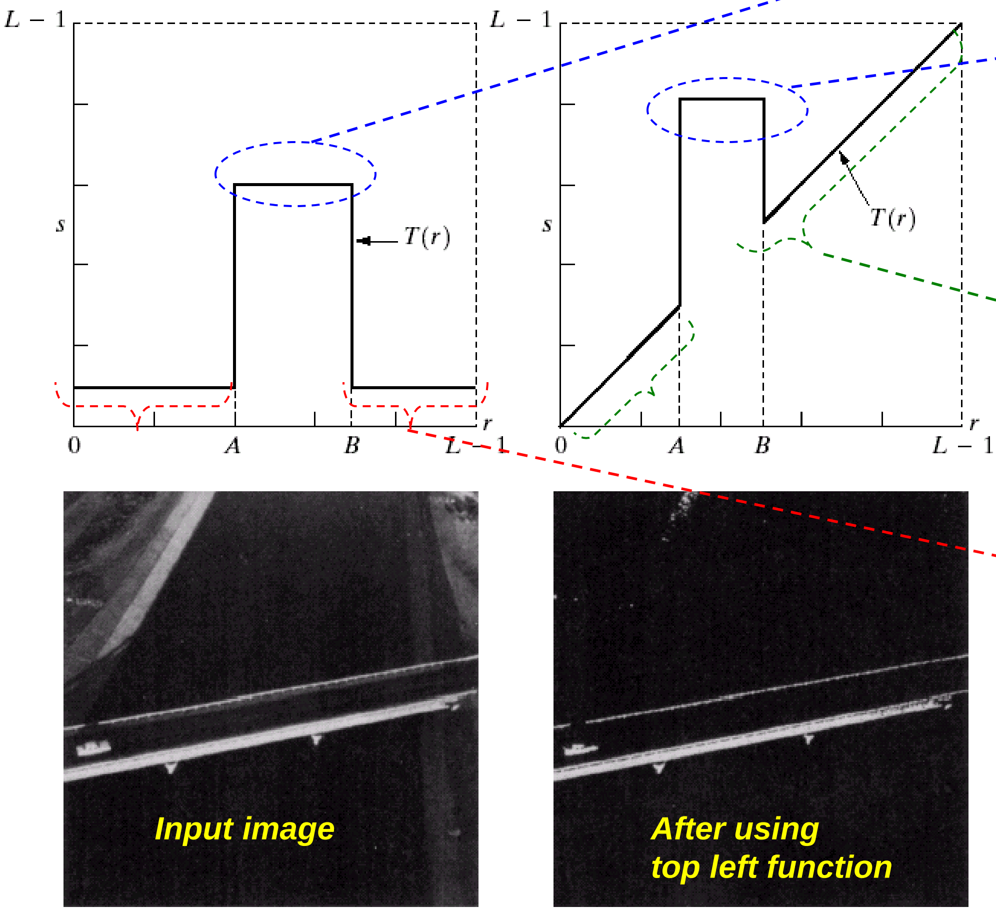
Image Enhancement in *Spatial Domain*

Piecewise-Linear Transformations – Gray-level Slicing

- *This technique is used to highlight a specific range of gray levels in a given image.*



Pixel values between [A, B] are highlighted

Other pixel values are preserved

Other pixel values are darkened

a b
c d

FIGURE 3.11 (a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

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Image Enhancement in *Spatial Domain*

Piecewise-Linear Transformations – Bit Plane Slicing

- ***Instead of highlighting gray-level ranges, highlighting the contribution made by each bit is useful and used in image compression.***

- ***Most significant bits contains the majority of the visually significant data.***

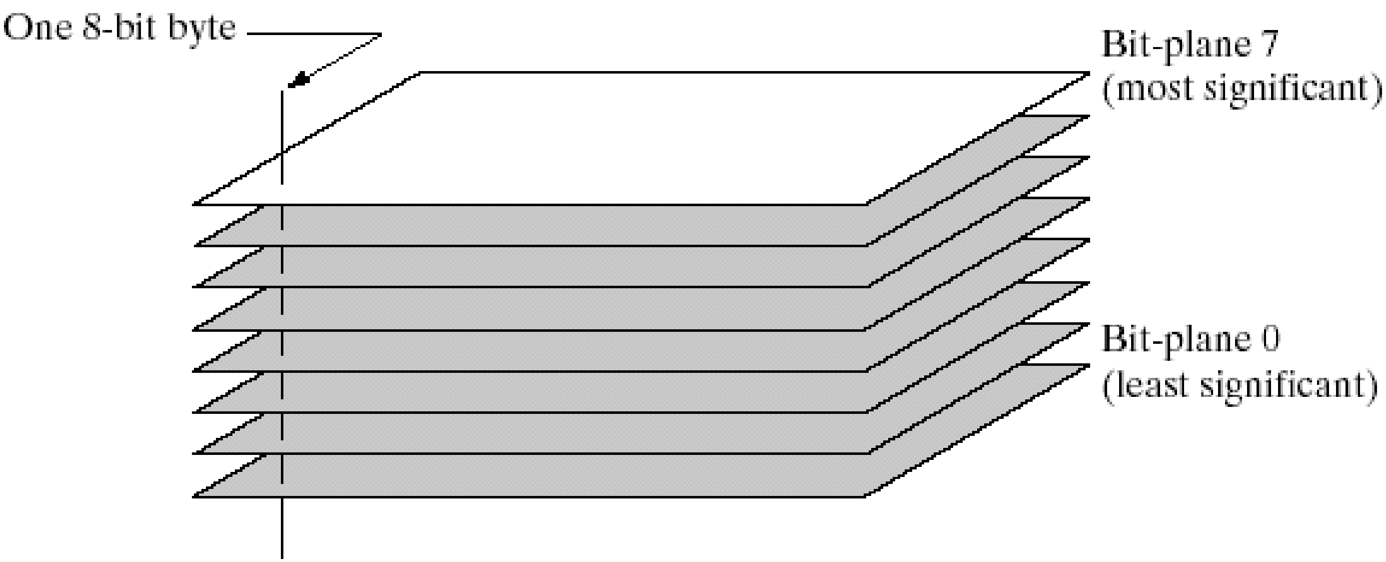

FIGURE 3.12 Bit-plane representation of an 8-bit image.

Image Enhancement in *Spatial Domain*

Piecewise-Linear Transformations – Bit Plane Slicing

Consider this 8-bit Fractal Image

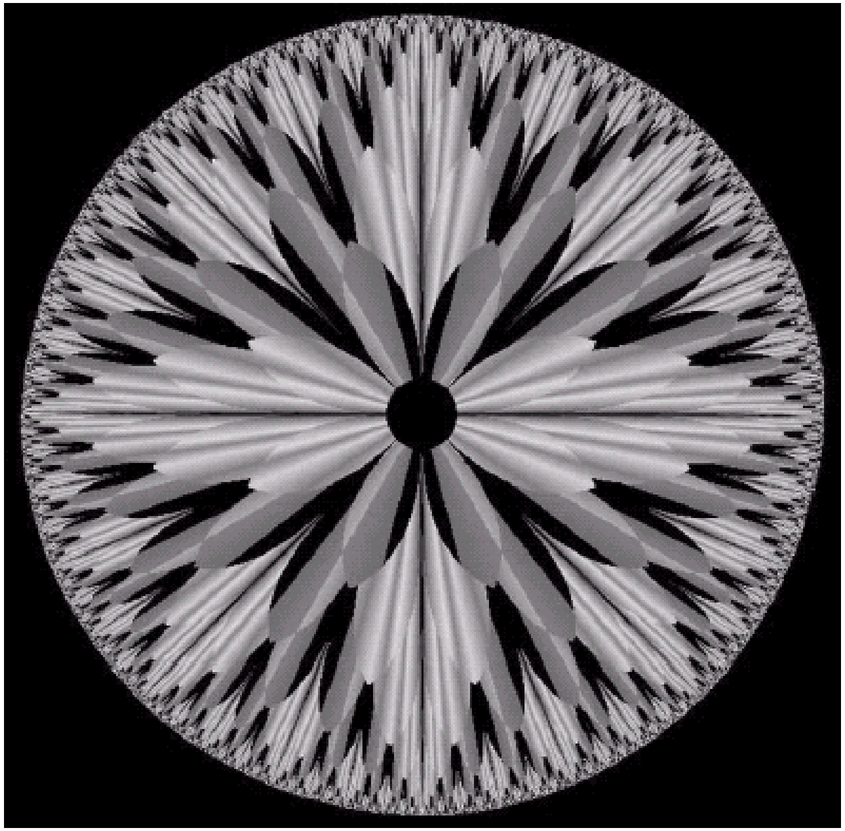

FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

Image Enhancement in *Spatial Domain*

Piecewise-Linear Transformations – Bit Plane Slicing

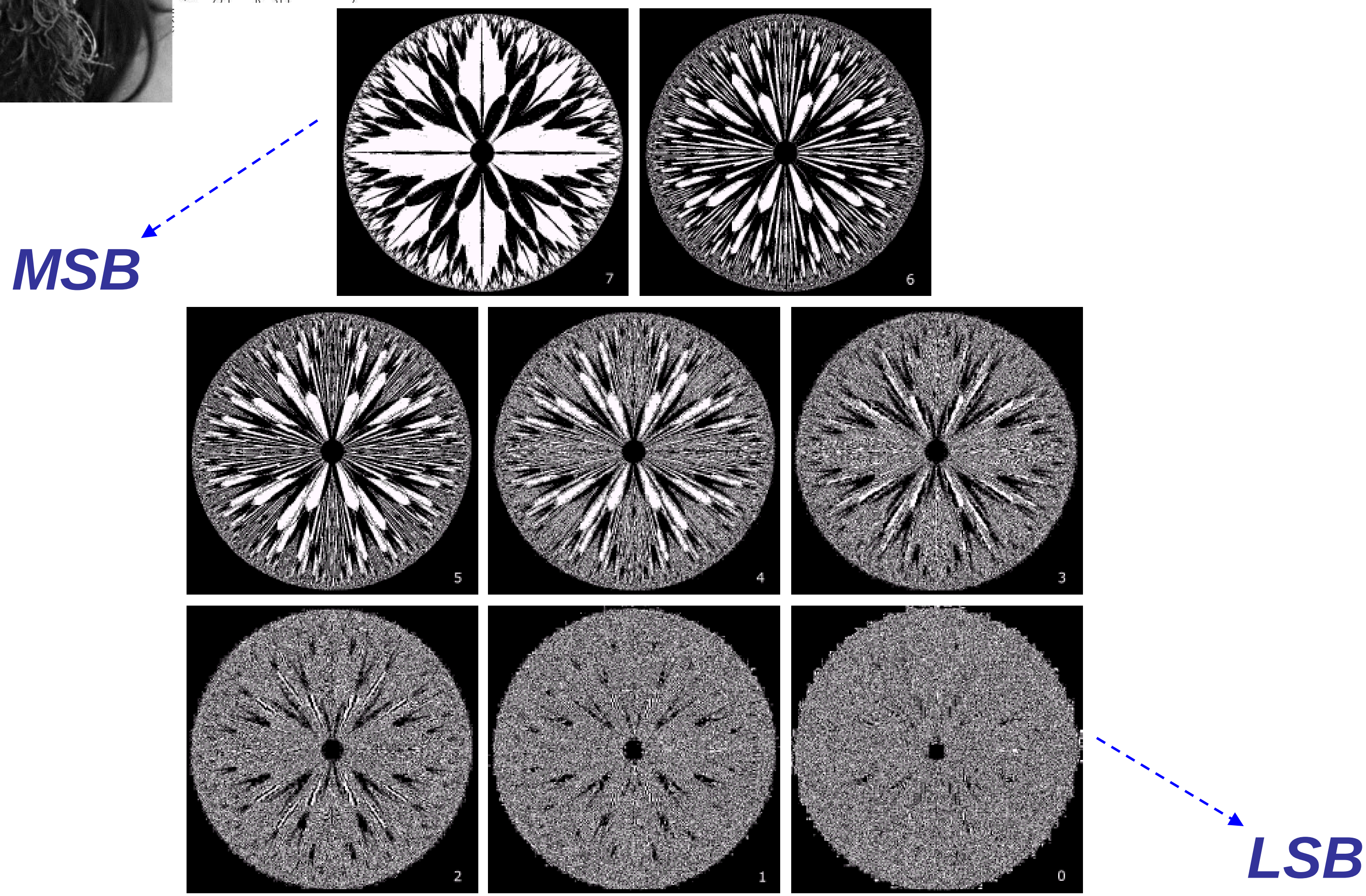

FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

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Image Enhancement in *Spatial Domain*

Histogram Processing

- *Histogram of a gray-level image in the range of [0, L-1] is the discrete function $h(r_k)=n_k$, where r_k is the k^{th} gray level and n_k is the number of pixels having gray level r_k.*

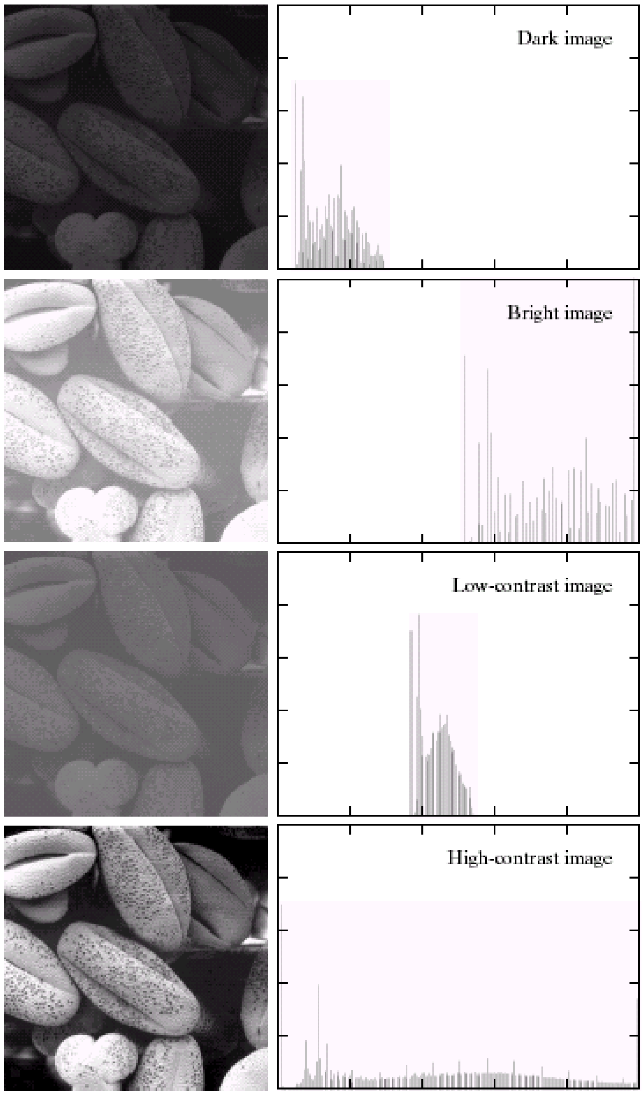
- *The histogram $h(r_k)$ can be normalized to $p(r_k)=n_k/n$, for $k=0,1,\ldots,L-1$.*

- *$p(r_k)$ can be considered to give an estimate of the probability of occurrence of ray level r_k.*

- *The cumulative of all components/probabilities of the normalized histogram is equal to 1.*

Image Enhancement in *Spatial Domain*

Histogram Processing



- ***Histograms of four basic gray-level characteristics.***
- ***The x axis shows the full range of the gray levels in the given image and the y axis corresponds to the number of occurrences of each gray $h(r_k)$ or the probability of a pixel to have that gray-level value, $p(r_k)$.***

Image Enhancement in *Spatial Domain*

Histogram Equalization

- *Histogram equalization is a method which increases the dynamic range of the gray-levels in a low-contrast image to cover full range of gray-levels.*
- ***Histogram equalization** is achieved by having a transformation function **T(r)**, which can be defined to be the Cumulative Distribution Function (CDF) of a given probability density function (PDF) of gray-levels in a given image.*

- *The histogram of an image can be considered as the approximation of the PDF of that image. Then the transformation function can be obtained by:*

$$s = T(r) = \int_0^r p_r(\omega)d\omega$$

- *where the intensity levels are considered as the continuous quantities normalized to the range [0,1] and $p(r_k)$ denotes the PDF of a given input image.*

Prepared By: *Dr. Hasan Demirel, PhD*

Image Enhancement in *Spatial Domain*

Histogram Equalization

•*The probability function of the output levels is **uniform**, that is:*

•*The transformation generates an image whose intensity levels are equally likely covering the entire range [0,1].*

• *The net result is the intensity-level equalization. Note that the transformation function is simply the CDF .*

Image Enhancement in *Spatial Domain*

Histogram Equalization

- ***Histograms are discrete quantities and we deal with probabilities and summations instead of PDFs and integrals.***

- ***Let $p(r_k)$ be the probability of occurrence of gray-level r_k in a given image:***

$$p(r_k) = \frac{n_k}{n} \quad k = 0, 1, 2, \ldots, L-1$$

- ***Then,***

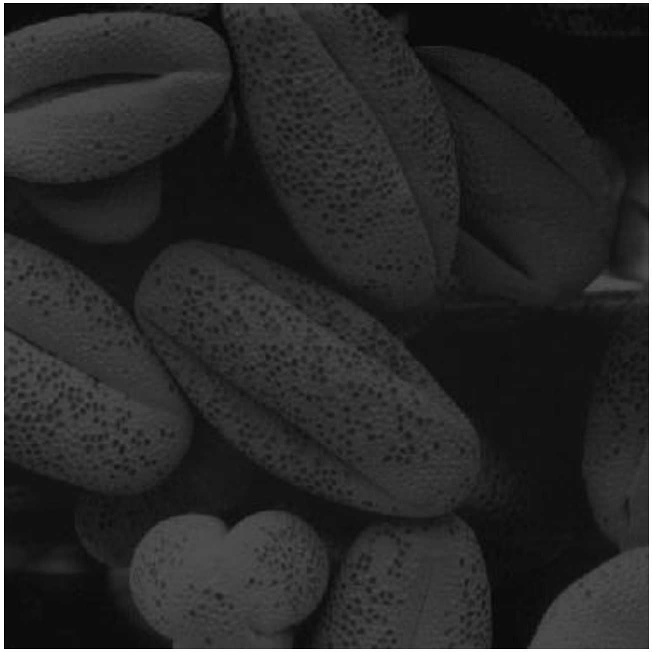
$$s_k = T(r_k) = \sum_{j=0}^{k} p(r_j)$$

$$= \sum_{j=0}^{k} \frac{n_j}{n} \quad k = 0, 1, 2, \ldots, L-1$$

Image Enhancement in *Spatial Domain*

Histogram Equalization

Low-contrast image



Corresponding histogram

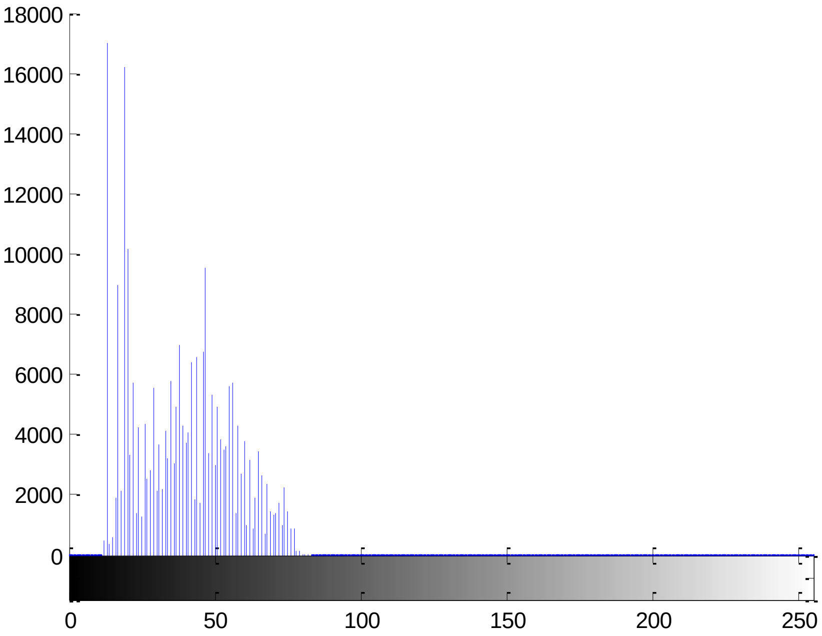
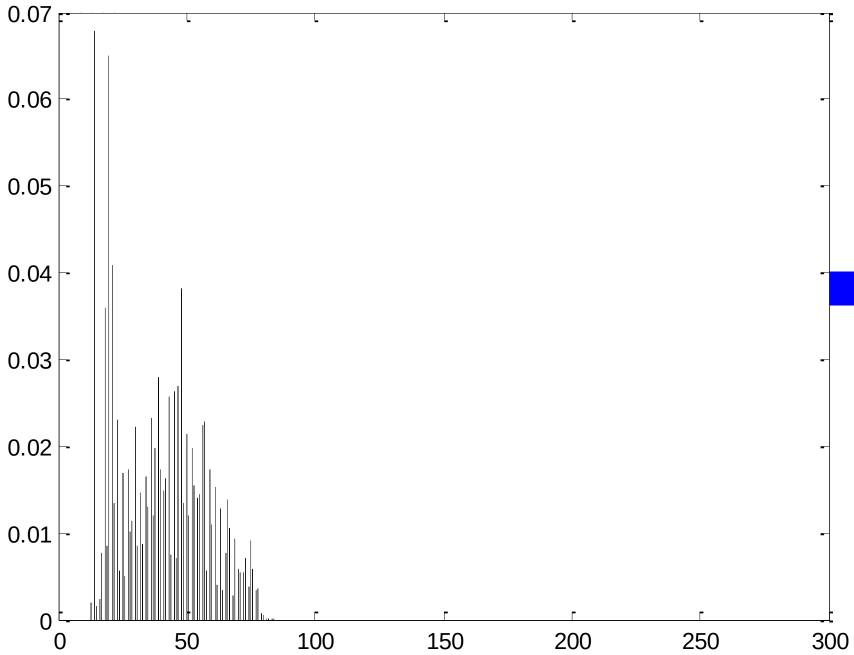


Image Enhancement in *Spatial Domain*

Histogram Equalization

Normalized Histogram of the input image (~ PDF)



Transformation function (~ CDF)

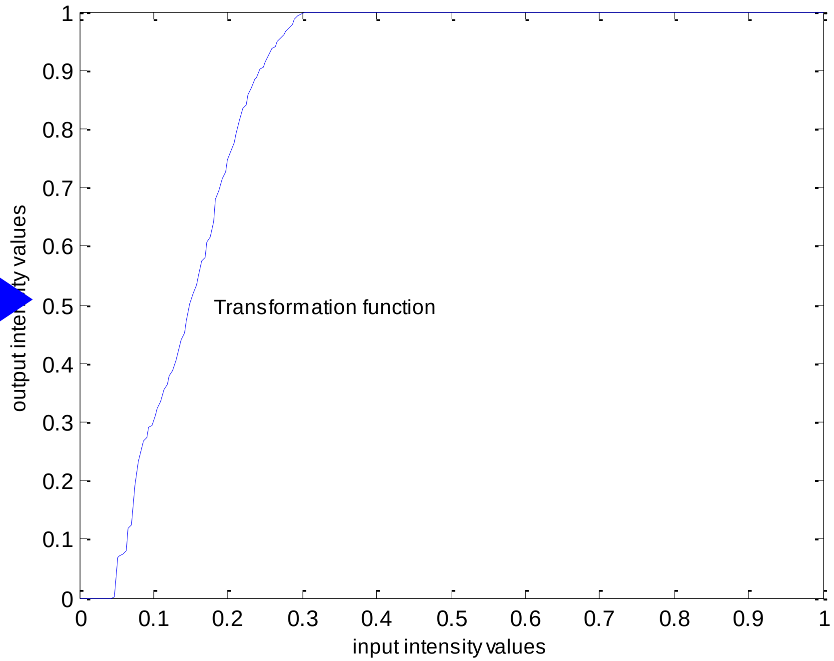
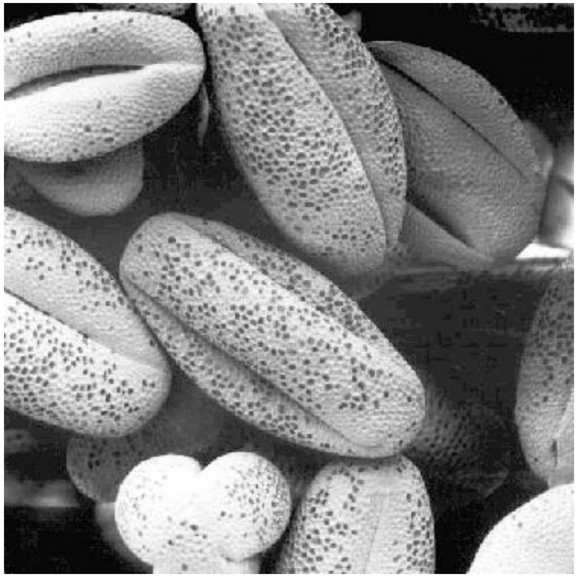


Image Enhancement in *Spatial Domain*

Histogram Equalization

Enhanced high-contrast image



Corresponding histogram

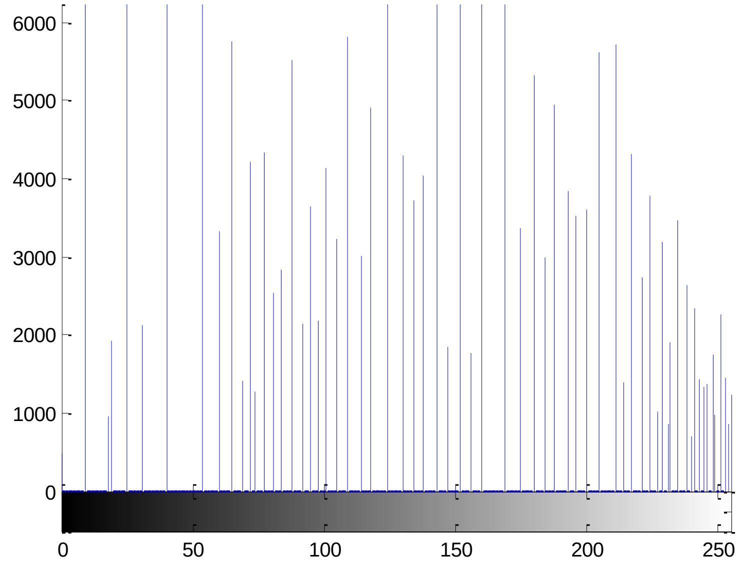
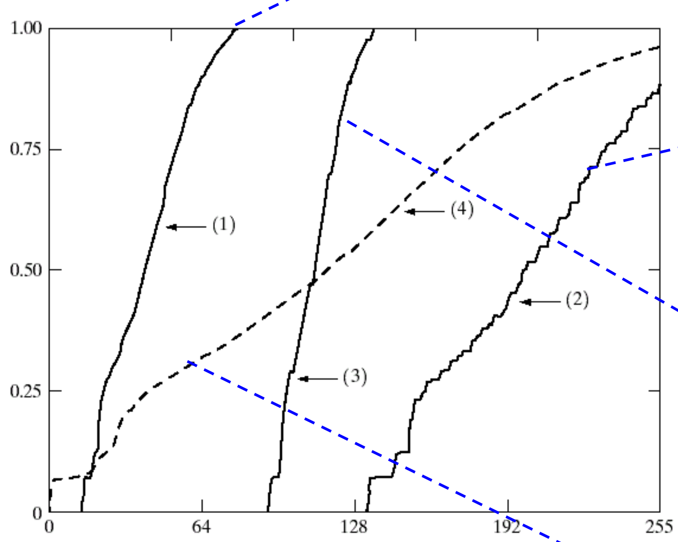
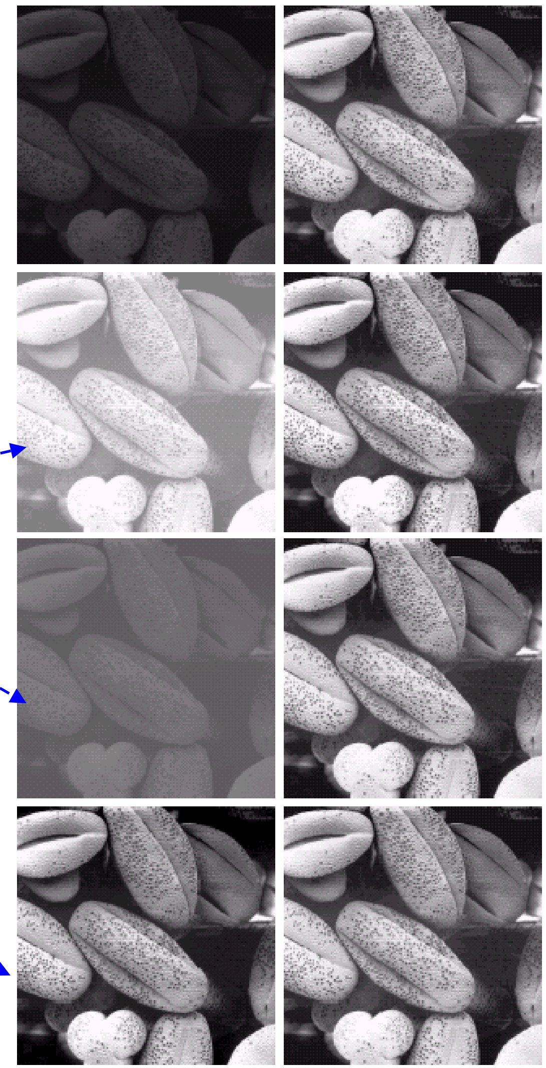


Image Enhancement in *Spatial Domain*

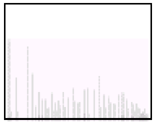
Histogram Equalization

FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).

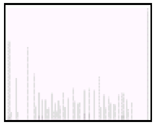




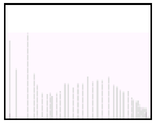
Dark image enhancement



Bright image enhancement



Low contrast image enhancement



High contrast image – almost no enhancement